CHAPTER 9

The Status of Categories and its Epistemological Stakes in the Fourteenth Century: The Case of Blasius of Parma

Joël Biard

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Blasius of Parma's *Questions on the Logical Treatises of Peter of Spain* contains, as one might expect, a treatise on categories. As the general title of the work indicates, this is not a systematic exposition of the subject, nor a literal commentary on Peter of Spain's text, but a series of questions, seventeen in all, which are either about general problems concerning predication, or about particular categories. In this treatise, Blasius develops a theory of categories which owes much to Ockhamism, be it directly or indirectly.

Blasius of Parma also wrote works on mathematics and natural philosophy. In natural philosophy he does not engage in a systematic logico-linguistic analysis of concepts and statements, as John Buridan had done half a century before. Nevertheless, a certain number of logical procedures are invoked and applied. In the case of the categories, his taking resort to logical procedures is usually occasioned by the problem of the status of the *significata* of the categories, that is the question of knowing if they signify something specific.

A reductionist conception of categories

Blasius' general conception of categories can be qualified as 'reductionist' in the sense that only two categories (substance and quality) have a direct and absolute signification. We do not find in his text any explicit statement on the point, as we do, for instance, in John Buridan, but it is clearly manifested by his treatment of quantity and relation.

Rejecting Walter Burley's alternative position, Blasius follows William of Ockham and John Buridan in taking categories in general to be terms, whose modes of signification and reference are to be analyzed, and not sorts of being. The term 'predicament' is a metalinguistic term, which designates a series of terms, ordered according to superiority and inferiority, from the most common to the less common, and predicable one of the other by essential predication.[1] The relation that is here indicated presupposes a precision Blasius has given previously, according to which "all that is predicated of another thing concerns, qua predicate, more things than that which is its subject qua subject".[2] This does not only imply a priority of direct predication over indirect or improper predication; it is clearly a matter of providing a purely extensional understanding of the relation between the different terms of the same categorial series. The term which is predicated in a proper and direct predication ought to refer to more things than the subject term; this is what marks its superior position in the categorial line[3]. This is also how Blasius interprets Porphyry, both in treatise III on categories and in treatise II (which contains only one question) on predicables.

The term 'predicament' is also used in another and complementary sense: since in each of these series, there is one term which is most general (for example 'substance'), it is that term which will be used to designate such a series of terms. Categories, then, are classes of terms, and each of them is characterized by the sort of question about primary substances that they can be used to answer, i.e. by their semantic properties:

1. Blaise de Parme, *Questiones super tractatus logice magistri Petri Hispani* (henceforward: *QTL*), III, 1, p. 227: "Predicamentum est sui termini debito modo ordinati secundum sub et supra, predicabiles de se invicem predicatione essentiali"; see also *QTL*, II, qu. un., p. 207: "Predicamentum est ordinatio terminorum secundum sub et supra de se invicem predicabilium essentialiter". Cf. the end of section 1 of Ashworth's essay, below.

2. *QTL*, II, qu. un., p. 207: "Omne quod de alio predicatur, ut predicatum est, in plus se habet quam illud quod subicitur sibi, ut subiectum est"; repeated a few lines below: "predicatum ut sic in plus se habet quam subiectum ut sic."

3. *QTL*, III, 2, p. 232: "Isti termini imaginandi sunt in una et eadem linea."

> The categories are divided according to the division of the terms which have different modes of signifying, connoting and asking .[4]

That is why, when Blasius raises the question, which had been unavoidable since Burley's commentary on *Categories*, whether categories are terms or things, he gives the following general answer:

> Only terms can be placed in a category and in no way external things, inasmuch as these are opposed to terms.[5]

As already indicated, these fundamental positions about the status of categories are not original: Blasius works within a widespread 14th-century tradition gathering Ockhamist and Buridanian teachings on that point.

On the other hand, Blasius does not explicitly raise the question of the number (the 'sufficiency') of categories, as he did for the Porphyrian 'predicables' in the lone question of Treatise II.[6] Neither does he divide the categories into those which have an absolute signification and those which have a connotative signification. We shall have to examine, in each case, the categories which are the most sensitive from this point of view.

Quantity: extension

The first one, of course, is the category of quantity. The problem with which we are concerned is treated in two steps. First, in the first question dedicated to quantity, question 5 of treatise III: "Is the quantified substance distinguished from its quantity, or is it the same thing as its quantity and its extension ? In other words, I seek to know behind these words if all quantity is a substance or a quality".[7] This initial formulation of the problem sees it from the

4. *QTL*, III, 15, p. 320: "Predicamenta sunt divisa secundum divisionem terminorum habentium diversum modum significandi, connotandi et interrogandi."

5. *QTL*, III, 2, p. 234: "solum termini sunt ponibiles in predicamento, nullo modo res extra distincte contra terminos."

6. *QTL*, II, p. 204 *sqq.* ; especially pp. 207-209.

7. *QTL*, III, 5, p. 263: "Utrum substantia quanta distinguatur a quantitate eius, vel

point of view of extension, or continuous quantity. Further on, the question is completed from the point of view of number, or discrete quantity.

In the first of these questions, Blasius answers in two steps, first according to a logical determination, then according to a physical determination. The logical determination is itself divided in two. First, the problem is treated on a metalinguistic level, as the identity of a term of the category of substance with a term of the category of quantity; secondly, as the identity of signified things. Only the second part is problematic and really pertinent. In that second part of the logical determination, Blasius lays down a series of conclusions about the identity or non-identity of quantity with substance or with quality.

Let us notice, by anticipation, what was indicated right from the title of the question: quality is here treated in the same way as substance. In other words, as in the Ockhamist doctrine, there are two categories whose absolute status is not questioned (that is to say that to a category of terms corresponds properly and directly a sort of things): substance and quality.

Paradoxically, Blasius does not tackle head-on the question of the real identity of a quality with its quantity or extension, although this question had been crucial for the conception of physical body since the thirteenth century and had become once more the object of attention due to the Ockhamist position and the debates to which it had given rise. Yet he takes it up when he takes into consideration the physical aspect of the question, and examines it at length in question 6 on book I of the *Physics*[8]. He there manifests caution, maybe because of the consequences for the sacrament of the Eucharist, which he mentions explicitly[9]. He successively develops both

idem sit quod sua quantitas et extensio ; sive queram sub his verbis : utrum omnis quantitas sit substantia vel qualitas."

8. *Quaest. Phys.*, 2ª lectura, ms. Vat. lat. 2159 (dated 1397), f° 71va-74rb.

9. See *Quaest. Phys.*, f° 74ra : during the discussion of a difficulty (knowing if the extended whiteness is identical to its extension), Blasius of Parma evokes the sacrament of the Altar as an argument in favour of the distinction between quality and extension : "vides quod in Sacramento Altaris est albedo et extensio, et non est dubium fieri non quanta, et hoc arguit quod ista albedo est extensa per extensionem sibi

positions. However, reductionist arguments are somewhat favoured.

In the logical determination of Question III. 5 of the *Tractatus logice*, the theses are general, although the examples are for the most part numerical. The first conclusion affirms clearly the real identity of the thing designated by a quantitative and by a qualitative term.

(1) Some quantity is a quality (*aliqua quantitas est qualitas*); the example is that of a whiteness of a foot. If we think of the consequences for the Eucharist[10] linked to the status of the quantity, it is clear that such a formulation makes the position of Thomas Aquinas impossible, according to whom a quantity is the subject of the qualities of transubstantiation.

(2) There is a quantity which is not a quality. The immediate meaning is simple: we take two qualities, their quantity (here the *binarius*, the number 2, or perhaps a pair) is not a quality because it is in no subject. This raises the question, to which we shall have to come back, of the status of mathematical terms, especially of numbers. The following conclusions only develop this initial intuition.

(3) There is a quantity that is neither a substance nor a quality.

(4) There is a quantity that is neither a substance nor substances, neither a quality nor qualities – we take as an example a pair formed by a substance and a quality.

I put aside conclusions (5), (6) and (7) and mention only conclusion (8), which comes back to the identity of a thing to which quantitative terms refer: a pair, *binarius*, can be the centuple of another *binarius*, if we think of two ants and two men. Here the centuple ratio is not between quantitative terms themselves (that would be a non-

condistinctam"; the fact is all the more noticeable as it is rare to see Blasius appeal to arguments of a theological nature.

10. It is precisely about the same example that Blasius evokes the sacrament of the Altar in the *Physics*.

sense) but between their referents. Therefore, quantitative terms do not supposit for anything but substantial (or qualitative) terms.

If we now look at what Blasius calls the 'physical determination', things are simpler. But is it here really about physics ? It is again about the theory of categories, considered from a metaphysical point of view, and with evident consequences in the field of natural philosophy. Blasius speaks here again without any nuance:

> Concerning the third article of the question, one must not doubt this conclusion: All quantity is a substance or a quality.[11]

The ontological basis is no less clear:

> Whichever thing is demonstrated, it is either a substance or an accident ... if it exists by itself, then it is a substance, if it is inherent in some other thing, then it is an accident.[12]

In fact, Blasius gives the arguments of the position that would admit a distinction between substance and quantity, he even grants that each position could be sustained with some persuasive force;[13] in fact, he seems to think that most of the authorities are in favour of this position,[14] and surely he is mainly thinking of Peter of Spain, on whose text he is commenting. However, even if this position may be rationnally supported, it implies paradoxical consequences, for example that we could remove extension and preserve Socrates, re-

11. "Pro tertio articulo questionis non est dubitandum de hac conclusione : omnis quantitas est substantia vel qualitas" (*QTL*, III, 5, p. 269).

12. .*QTL*, III, 5, p. 269: "quacumque re demonstrata, ipsa est substantia vel accidens ... Si est per se existens, sic est substantia, si est alteri inherens, sic est qualitas" The unrestricted range of this thesis and its status as a principle are underlined: "Immo habetur pro principio in quacumque Facultate quod omne quod est aut est substantia vel accidens".

13. *QTL*, III, 5, p. 269: "Et quia ista materia est sustentabilis pro utraque parte ...".

14. *QTL*, III, 5, p. 269 : "... una opinio ponit hanc conclusionem : substantia quanta non est sua quantitas vel extensio .... Ista tamen videtur esse vera propter ... multas auctoritates Aristotelis et aliorum sapientium".

move corporeity and preserve a man, and still more curious things[15]. Furthermore, he claims, it has the defect of resting on the methodological principle of divine omnipotence in order to attest the separability *de jure* of naturally inseparable things.

He exposes more briefly the position which identifies substance and quantity, drawing above all the consequences: we have to admit that one and the same *substantia quanta* can be under different extensions, smaller or bigger, because of a local movement. This thesis, formulated in typically Ockhamist terms, implies that extension is considered as a disposition of the substance, and not as a quantity, which would be really distinct; the only means of thinking an extensional variation of one and the same substance is to reduce it to a movement of its parts[16].

Quantity: number

From Thomas Aquinas to William of Ockham, the connection between substance and extension was at the centre of the discussion about the status of quantity. That is why, in chapter 44 of the first treatise of his *Summa logicae*, William of Ockham briefly mentions discrete quantity and dedicates most of his argumentation to the connections between the point, the line or the surface, or more generally between continuous quantity and substance or quality[17].

Blasius of Parma, for his part, writes at greater length about number. There is here, without any doubt, an evolution which is characteristic of his treatise. First, he makes number the subject of a special question (III.9), which is rather short, but whose formulation explicitely raises the problem of 'reduction': "Is number the

15. See more details in Biard 2003a.

16. In the initial arguments, Blasius introduced several explanations on the sphericity of a piece of wax, in order to prove that this sphericity is different from the substance itself. Here, it would thus be suitable to refute these arguments, which were going in the direction of the other position, however "probabilis". He, nevertheless, implies that their refutation does not cause any major problem: "Si tamen quis velit tenere partem oppositam, iudicat per se ad eas" (*QTL.*, III, 5, p. 271).

17. See William of Ockham, *Summa logicae*, p. 132-139.

things which are numbered, or is it distinguished from them?"[18] The arguments in favour of a distinction rest especially on the possibility of numbering and counting regardless of the real state of things, while the argument against the being of number takes up a process applied elsewhere to the point, namely the impossibility of assigning to it the status of substance or of accident. As for the solution, after having mentioned, but also contested, the distinction between numbering number, numbered number and number by which we number[19], Blasius establishes, this time without any hesitation or nuance, the identity of number and of things numbered: "Number is the very things that are numbered."[20] This identity is clearly founded on the identity of reference (supposition) of the terms in a proposition such as 'These ten horses are a number' (*isti decem equi sunt numerus*). So a numerical term or concept does not refer to something distinct, which would have a specific mode of being or subsisting. Blasius however states that if we understand by 'number' the words or the concepts by which we count, then there is not identity, for then number is an accident of the soul.

Secondly, the importance of the question of number, relatively to that of extension, stands out owing to the fact that Blasius has dedicated a good part of question 5 to the identity of number and things numbered, although it ought to be dedicated to extension. Indeed, conclusions 2 to 8 (i.e., all the conclusions save one), concern numbers. In a previous article[21], I did not sufficiently realize the importance of this point and the significance of these conclusions. First they affirm the non-identity, in a certain sense, of quantity and quality. In conclusions 2 and 3, the quantity (the *binarius*), is not a quality nor a substance, because it is qualities (ie. *several* qualities). The formulations "There is a quantity which is not a quality" (*aliqua est quantitas que non est qualitas*), "There is a quantity which is not a substance nor a quality" (*aliqua est quantitas que non est substantia nec*

18. "Utrum numerus sit res numerate vel distinguatur ab eis" (*QTL.*, III, 9, p. 289-292).
19. *QTL.*, III, 9, p. 291 : "Ex istis conclusionibus evidenter apparet quod distinctio premissa posita tam ab antiquis quam a modernis est nullius valoris".
20. "Numerus est ipse res numerate" (*QTL.*, III, 9) ; cf *Quaest. Phys.*, I, 6, f° 72vb.
21. See foonote 15.

qualitas)[22] allow the possibility of some conceptual independence of what is signified by the subject, but these theses assimilate the reference of these terms either to one or several qualities (conclusions 1, 2 and 3), or to one or several substances, material or immaterial, or even to one quality and one substance (conclusion 4)[23].

Nevertheless, in such cases, we shall be able to realize operations on numbers only by attributing to them a certain unity, even a signification which cannot totally be reduced to reference. The point is not examined in depth, but the last two conclusions already progress in this direction. The 7th shows that if we reduce number to its reference, some numbers will be neither equal nor unequal (*e.g.* two men and two intelligences). The 8th, in a complementary manner, shows that in the same perspective, a *binarius* could be a hundred times bigger than another (two men and two ants). Either of these hypotheses would make arithmetic impossible. But, for Blasius, the value of mathematics (arithmetics, geometry, theory of proportions) is a fact, and so must be accounted for. We cannot, therefore, limit ourselves to this point. In order to overcome this difficulty, we have to consider, more briefly, another category, that of relation, and then reflect on its use in the field of mathematics.

Relation

Blasius dedicates three questions to relation. The last, qu. 14, asks

> Whether relation is something distinct from the things related and designated by the terms of the category *ad aliquid*, i.e., I wish to ask in this question whether fatherhood is different from the thing which is the father, and whether dependence is something different from that which is dependent.[24]

22. . See *QTL*, III, 5, p. 267.

23. . See *QTL*, III, 5, p. 267-268 ; we find the same formulations in *Quaest. Phys.* I, 6, f° 72vb.

24. . *QTL*., III, 14, p. 314: "Utrum relatio sit res distincta a rebus invicem relatis et importatis per terminos de predicamento ad aliquid, ut velim querere in illa questione utrum paternitas sit ista res que est pater vel distincta a patre, et dependentia sit res distincta a dependente".

The treatment is rather short, and the solution establishes without ambiguity that "relation is not a thing distinct from the things signified by the terms of the category *ad aliquid*", so that "dependence is the dependent thing itself", and that relation is "the things related to each other" (*ipse res invicem relate* – I correct the edition p. 316), except in the case of terms which do not refer to external things, i.e. metalinguistic statements, in which the terms themselves are the significates. The only justification of his claim is a reference to an argument *ad oppositum*, according to which the position of such a dependence would imply an infinite process. This conception of relation would be made explicit and unfolded in Blasius' mathematical texts.

To summarize, Blasius develops, in a perspective that is close to that of Ockham, a reductionist conception of categories in which quantity and relation are reduced to substances and qualities. However, we must notice that, concerning quantity, he does not spend as much time on extension, a question which is decisive for the status of material body, as he does on number.

Epistemological stakes

If we now take into consideration other texts of Blasius of Parma, and in particular his *Questions on Thomas Bradwardine's Treatise on Proportions*, we see that his conception of categories and the implied ontology, are related to a conception of the status of mathematics, and of the relation between mathematics and natural philosophy. To make this clear, we must start from the category of relation.

Indeed, the ratio (in latin *proportio*), which is the subject of Bradwardine's treatise, and then of Blasius' questions, is defined as a *habitudo*. Question 2 discusses the following definition:

> Consequently, in the second question, it is asked whether a ratio is, properly speaking, the relation (*habitudo*) of two quantities to each other.[25]

25. Blaise de Parme, *Questiones circa tractatus proportionum magistri Thome Braduardini* [henceforward: *QTP*], qu. 2, p. 61: "Consequenter secundo queritur utrum proportio proprie dicta sit duarum quantitatum unius ad alteram habitudo."

The term *habitudo* is a classical one. We find it in the version of the *Elements* of Euclid, composed by Campanus of Novara in the 13th century,[26] and it was taken up by Thomas Bradwardine.[27] In his *Questions on the Meteorologica*, Blasius attributes this definition to Euclid.[28] The formulations clearly indicate a relation of something to another thing, *unius ad alteram*. During the argumentation, Blasius considers a consequence as another example of *habitudo*, and further on he evokes a comparison. Indeed, this definition is not the real object of the question, contrary to what the title suggests. The question is "Which thing is a ratio ?" (*Que res est proportio?*), and this second formulation introduces a more ontological interrogation about the connection between the relation itself (more precisely, the mathematical ratio as relation) and the things put in relation. The main conclusion is the following: "A ratio is things which are related to each other",[29] and this is affirmed again in the reply to the contrary arguments.[30]

At that stage, the ratio, one of the central objects of the mathematical theory of the period, and furthermore for the mathematization of physical phenomena, is characterised in a way which makes use of the reductionist conception formulated in logic. We find the same process in another question, question 4, about the ratio between the diagonal and the side of the square, and in this case such a reduction is presented, in the case of geometrical concepts, with an interesting accuracy:

26. See H. L. L. Busard 2005 (*Campanus of Novara*), p. 103, df. 3: "Proportio est duarum quantaecunque sint eiusdem generis quantitatum, certa alterius ad alteram habitudo."

27. Thomas Bradwardine, *Tract. de proportionibus*, p. 66, l. 8-10 : "Proportio autem quae proprie est accepta, in solis quantitatibus reperitur. Quae definitur hoc modo : Proportio est duarum quantitatum eiusdem generis unius ad alteram habitudo."

28. *Quaest. metheororum*, I, qu. 3, ms. Vat. Chigi O. IV. 41, f° 61va-vb: "Dico primo quod proportio est duarum quantitatum alterius ad alteram certa habitudo, et hec diffinitio habetur ab Euclide V° *Elementorum* et a Thoma Barduardino."

29. "proportio est res invicem proportionate" (*QTP*, 2, p. 63).

30. *QTP*, 2., p. 65.

> I say that if you intend to speak like philosophers, who say that lines are not distinguished from surfaces, nor surfaces from bodies, then it must be conceded that the diameter of the square is the square itself and that the side of the square is also the square itself.[31]

This language (or this mode of thinking) is therefore that of 'philosophers' – and we must probably understand by that expression 'natural philosophers', particularly physicists, while also allowing it a larger scope since logic and ontology are concerned.

Geometrical concepts are situated at a level which is different from the categories of substance or quantity, or even of number and things numbered. We can, however, detect the same tendency to reduce some concepts, considered as having no proper and direct reference, to a 'thing' to which all the concepts of that series refer, namely body. Surely, 'body' is still a mathematical concept, pertaining to continuous quantity, but we could again ask about it the question about the reduction of extension to substance. We are indeed engaged in the same process, the same approach, even if we stop, here, at a stage which is not the last.

Another interesting precision is the outlined dissociation between "speaking like philosophers" (*loqui ut philosophi*) and "the way of mathematicians" (*modus mathematicorum*).[32] The first approach leads to reduction, logical as well as metaphysical; the second imagines for example lines that are indivisible according to width:

> But different is the way of mathematicians who imagine lines which are indivisible according to width.[33]

But we must not believe that this approach is to be depreciated, condemned in the name of philosophy: we are in a treatise where Blasius discusses the properly mathematical value of this or that

31. *QTP*, 4, p. 86. "... dico quod si intendis loqui ut philosophi, dicentes lineas non distingui a superficiebus nec superficies a corporibus, tunc erit concedendum quod dyameter quadrati erit ipsum quadratum, et idem de costa ipsius quadrati."

32. This point is examined in greater depth in Biard, 2003b.

33. *QTP*, p. 86: "Sed alius est modus mathematicorum ymaginantium lineas indivisibiles secundum latitudinem."

definition. On the one hand, consequently, Blasius remains true to his logical and metaphysical conceptions, recalling that the determinations of quantity, numbers or extensions, do not signify things that would be distinct; from that point of view, consequently, numbers must be assimilated to things that are numbered, lines and surfaces to bodies, ratios to the things related. Blasius refuses to hypostatize mathematical objects. On the other hand, he needs to confer a certain validity on mathematical conceptual instruments, in particular on the theory of ratios. Indeed, the fact of limiting ourselves to the point of view which combines elements of logic, physics and philosophy would not only remove all ontological consistency from the ratio, but would also lead to paradoxes, impossible to sustain mathematically. In particular, since the ratio is reducible to things which are related, the ratio between the numbers 2 and 3, for example (or, if we go to the end of the reduction, between 2 things of some sort and 3 things of another sort[34]), would be the same as the ratio between 3 and 2, or the double ratio (*proportio dupla*) would be the same as the subduple ratio (*proportio subdupla*), and more generally "the same thing is a ratio of a greater inequality and a ratio of a smaller inequality".[35] This opposition between the mathematical and the philosophical way of speaking is underlined in question 3 of the *Questions on the treatise on proportions*, which must be read in connection with the previously mentioned passages from the *Questions on logical treatises*:

> when arithmeticians speak of number, they distinguish number from things which are numbered, and in no way do they consider numbered things, but natural philosophers take number for numbered things.[36]

34. Which implies that the word 'ratio' should be used in a broad sense, and not in a narrow sense as the ratio between two quantities of the same sort ; but such a broader use is allowed.

35. *QTP*, 2, p. 64; The expression 'the same thing is' underlines the identity of reference ; a ratio A/B is of greater inequality if A is superior to B, and of lesser inequality if B is superior to A. See also Biard & Rommevaux, « Introduction » to *QTP*, p. 18.

36. *QTP*, 3, p. 70: "... dum arismetrici loquuntur de numero, distinguunt numerum

So they do not reduce number to things and treat it, we may say, as an 'object' to which they give some autonomy. For if mathematicians were to proceed "as philosophers", mathematics would be emptied of all content. The point is formulated through an objection, which threatens the very existence of a treatise on ratios:

> We cannot say that the relation (*habitudo*) is things which are related to each other, as are a man and a donkey, since then a relation would be nothing, as nothing is a man and a donkey.[37]

Being neither a substance (since only the singular is a substance), nor an accident (since an accident could not be subjectively in two distinct substances), the ratio would be nothing, would not be a thing.

In question 5, Blasius in the reply to an objection comes to characterize the *modus mathematicorum:*

> I say that if one conceives of a ratio as things which are related to each other ... the antecedent of the argument must be conceded. ... But if one conceives of a ratio according to its formal reason, the antecedent must be denied.[38]

Mathematicalia are not treated as independent substances, as they might be by Platonists, but the formal reason, that is to say the active mode of conceiving, becomes the proper object of the mathematician, and is treated and handled as such. The same procedure is

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contra res numeratas, et nullo modo considerant de rebus numeratis, sed philosophi naturales capiunt numerum pro rebus numeratis."

37. *QTP*, 2, p. 62-63: "Non potest dici quod habitudo est res invicem proportionate sicut sunt homo et asinus, quia tunc nichil esset habitudo, sicut nichil esset homo et asinus"; at the end of the sentence, we must understand 'nothing' (nihil) as 'not a thing'; this brings us back to a position which is discussed as much in Buridan as in the condemnations of Nicolas of Autrécourt.

38. *QTP*, 5, p. 94: "Dico quod capiendo proportionem pro rebus proportionatis ... concedendum est tunc antecedens rationis. ... Sed capiendo proportionem secundum eius rationem formalem, antecedens est negandum." See also p. 91 : "et quia iste modus loquendi est inconsuetus, loquar nunc de proportione secundum rationem formalem ...".

applied, as we have seen, to lines and surfaces, as it is to the question of indivisibles and of the continuous.[39]

This duality of point of view is not proper to the *Questions on the Treatise on Proportions*. We find it again in many other texts by Blasius of Parma. Thus taking a point to be indivisible is proper to the mathematician, while from a physical point of view everything is divisible. In his *Questions on Generation and Corruption*, Blasius shows the contradictions that would result from the position (the 'imagination', as is the term used for mathematics) of physical indivisibles;[40] similarly the question "Does the sphere touch the plane in one point?"[41] shows the contradictions which would result from transferring the mathematical concepts of point, line and sphere to natural philosophy. On the other hand, in mathematics, Blasius is more cautious concerning composing a continuum out of indivisibles.[42] The mathematical concept of point is indeed an imagined indivisible, while the physicist takes it rather as an infinitely small thing.[43]

This opposition between mathematical and physical concepts is particularly clear in the *Question on the Contact between a Sphere and a Plane*. Blasius appears very dependent on a certain Buridanian tradition, as much for the definitions of the point as for the general direction of the question towards a problem in the epistemology of mathematics. But what is new, is that Blasius clearly distinguishes the physical treatment and the mathematical treatment of the question (even if the details are sometimes confused). There, Blasius develops once more a reductionist ontology for the term 'contact', which is a relational term:

> I presuppose first, as it is true in the things, that the contact of bodies is the bodies touching each other.[44]

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39. See *QTL*, III, 10, p. 293-298.

40. . *Quaest. de gen. et corr.*, I, 15, ms. Vat. Chigi O IV 41, f° 23va.

41. "Utrum spera tangit planum in puncto". See Biard & Rommevaux 2009, which contains an introduction, the edition of the question and a French translation.

42. See Biard 2009.

43. We find clear allusions to these different conceptions in question 11 of the *Questiones circa tractatum proportionum*, ed. cit., p. 192.

44. "Et presuppono primo, ut est rei veritas, quod tactus corporum est corpora sese

Above all, he develops the thesis according to which mathematical concepts do not have real referents, at least not proper and absolute referents. For natural philosophy, the fact of admitting the being of the point, the line and the surface would generate contradictions. After having set forth such contradictions, which result from the parallel between the concepts of point, line and surface on the one hand, and his reductionist ontology on the other, Blasius announces that he will determine the question "first physically and then geometrically". Physically speaking, all geometrical conclusions are false, since they consider something which does not exist. Nevertheless, these statements make sense if we understand them *conditionaliter* or *ex suppositione*. I shall not here go into the details of this treatment, which combines logical and mathematical considerations,[45] From a purely logical point of view, one could try to reduce the problem to a question of connotation and syncategoremata. But Blasius' aim is not such a semantical reduction. On the contrary, by autonomizing this mode of conceiving, this *ratio formalis*, he aims to produce by means of the imagination some *mathematicalia* which can be manipulated as such, opening the conceptual space in which mathematics may be unfolded.

Conclusion

The logical and ontological basis of the study of categories is not so far from the Ockhamist doctrine: two categories of absolute terms, and a strong reductionist approach to such categories as quantity and relation. We have noticed, however, that in comparison to that model, the general balance is slightly modified. On one hand, in the treatment of quantity, the place dedicated to number is as important as the one dedicated to extension. On the other hand, the category of relation is presented, not from the perspective of theological problems, but from the perspective of the mathematical theory

tangentia." Blasius will come back several times to this point in the course of the question.

45. All this has been set forth in details in our *Introduction* to the edition quoted above (footnote 41).

of ratios and proportions. So Blasius of Parma proposes an epistemology of mathematics which takes up some suggestions made by Buridan or made in texts close to those of Buridan. But the place given to it is definitely more prominent, in accordance with the general orientation of Blasius' works – let us not forget that in Italy his *Questions on the treatise on proportions* were discussed in natural philosophy until the 16th century.

These developments probably show the impossibility of maintaining a purely extensional vision of the signification of concepts if we want to give sense to mathematics – which is a reasonable aim. Blasius does not turn to a mathematical Platonism, as was often to happen after the translation of Proclus' commentary on book I of Euclid's *Elements* in the 16th century. But he does not stick with abstractionist statements, as was frequently the case in the beginning of the 14th century. The autonomization of the formal reason (which is not a concept abstracted from sensible things) allows, without treating mathematical substances as real beings, to unfold mathematics at its own level of being without regard to any logical or philosophical theses that might be incompatible with its results.

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